Date: 18/05/2025

Time : 3 hrs.

Answers & Solutions

for

JEE (Advanced)-2025 (Paper-2)

Max. Marks: 180

PART-I : MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x.

Then which one of the following statements is TRUE?

(A) For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(B) For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

(C) For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(D) For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Answer (C)

Sol. $e^{x_0} + x_0 = 0$ (given)

For $\alpha = 2$, $g(x) = \frac{3x(e^x + 1) - 2(e^x + x)}{3(e^x + 1)}$

$= x - \frac{2}{3}\left(\frac{e^x + x}{e^x + 1}\right)$

$\Rightarrow l_1 = \lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right|$

$= \lim_{x \to x_0} \left| \frac{x - \frac{2}{3}\left(\frac{e^x + x}{e^x + 1}\right) + e^{x_0}}{x - x_0} \right|$

$\because e^{x_0} = -x_0$

$\Rightarrow l_1 = \lim_{x \to x_0} \left| \frac{(x - x_0) - \frac{2}{3}\left(\frac{e^x + x}{e^x + 1}\right)}{x - x_0} \right|$

$= \lim_{x \to x_0} \left| 1 - \frac{2}{3}\left(\frac{e^x + x}{e^x + 1}\right) \cdot \frac{1}{x - x_0} \right|$

Let $l_2 = \lim_{x \to x_0} \frac{\left(\frac{e^x + x}{e^x + 1}\right)}{x - x_0}$, form : $\frac{0}{0}$

$\Rightarrow l_2 = \lim_{x \to x_0} \frac{(e^x + 1)(e^x + 1) - (e^x + x)(e^x)}{(e^x + 1)^2} = 1$

$\Rightarrow\ l_1 = \left|1-\frac{2}{3}\right| = \frac{1}{3}$

For $\alpha = 3,\ g(x) = \frac{3x\left(e^x+1\right)-3\left(e^x+x\right)}{3\left(e^x+1\right)}$

$= x - \frac{e^x+x}{e^x+1}$

$l = \lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|$

$= \lim_{x\to x_0}\left|\frac{x-\left(\frac{e^x+x}{e^x+1}\right)+e^{x_0}}{x-x_0}\right|$

$\because\ e^x = -x_0$

$\Rightarrow\ l = \lim_{x\to x_0}\left|\frac{\left(x-x_0\right)-\frac{e^x+x}{e^x+1}}{x-x_0}\right|$

$= \lim_{x\to x_0}\left|1-\left(\frac{\frac{e^x+x}{e^x+1}}{x-x_0}\right)\right|$

Let $l_2 = \lim_{x\to x_0}\left(\frac{\frac{e^x+x}{e^x+1}}{x-x_0}\right)$, form: $\frac{0}{0}$

$\Rightarrow\ l_2 = 1$

$\Rightarrow\ l = |1-1| = 0$

2. Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$\left\{(x,y)\in\mathbb{R}\times\mathbb{R} : x>0, y>\frac{1}{x}, 5x-4y-1>0, 4x+4y-17<0\right\}$

is

(A) $\frac{17}{16}-\log_e 4$

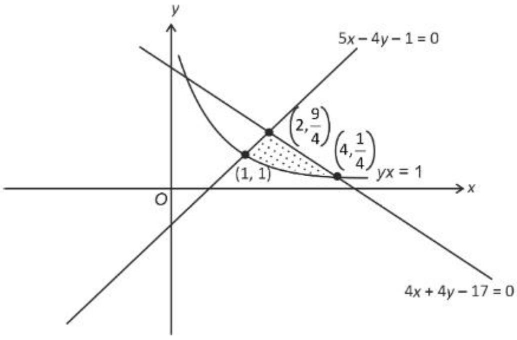
(B) $\frac{33}{8}-\log_e 4$

(C) $\frac{57}{8}-\log_e 4$

(D) $\frac{17}{8}-\log_e 4$

Answer (B)

Sol.



Area = $\int_1^2 \left(\frac{5x-1}{4}-\frac{1}{x}\right)dx+\int_2^4\left(\frac{17-4x}{4}-\frac{1}{x}\right)dx$

$=\frac{33}{8}-\ln 4$

Option (B) is correct.

3. The total number of real solutions of the equation

$\theta=\tan^{-1}(2\tan\theta)-\frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$ is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, respectively).

(A) 1

(B) 2

(C) 3

(D) 5

Answer (C)

Sol. Let $\frac{\tan\theta}{3}=\alpha \Rightarrow \tan\theta=3\alpha$

$\theta=\tan^{-1}3\alpha$

$\tan^{-1}(3\alpha)=\tan^{-1}(6\alpha)-\frac{1}{2}\sin^{-1}\left(\frac{18\alpha}{9+9\alpha^2}\right)$

$\tan^{-1}(3\alpha)=\tan^{-1}(6\alpha)-\frac{1}{2}\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$

$\frac{1}{2}\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)=\tan^{-1}(6\alpha)-\tan^{-1}(3\alpha)$

$=\tan^{-1}\left(\frac{6\alpha-3\alpha}{1+18\alpha^2}\right)$

$$\sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)=2\tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$$

$$\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2}\right)=2\tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$$

$$\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2}\right)-\tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)=\tan^{-1}\left(\frac{3\alpha}{1+18\alpha^2}\right)$$

$$\frac{\frac{2\alpha}{1-\alpha^2}-\frac{3\alpha}{1+18\alpha^2}}{1+\frac{(2\alpha)(3\alpha)}{(1-\alpha^2)(1+18\alpha^2)}}=\frac{3\alpha}{(1+18\alpha^2)}$$

$$\Rightarrow \quad \frac{2+36\alpha^2-3+3\alpha^2}{-18\alpha^4+17\alpha^2+1+6\alpha^2}=\frac{3}{1+18\alpha^2}$$

Let $\alpha^2 = t$

$$\frac{32t-1}{-18t^2+23t+1}=\frac{3}{1+18t}$$

$\Rightarrow \quad t^2(702 + 54) + t\,(21 - 69) - 1 - 3 = 0$

$756t^2 - 48t - 4 = 0$

Product of roots < 0

$\Rightarrow$ Only two values of $x \neq 0$

4. Let S denote the locus of the point of intersection of the pair of lines

$4x - 3y = 12\alpha,$

$4\alpha x + 3\alpha y = 12,$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4x - \frac{3}{\sqrt{2}}y = 0$.

Then the value of pq is

(A) $-6\sqrt{2}$

(B) $-3\sqrt{2}$

(C) $-9\sqrt{2}$

(D) $-12\sqrt{2}$

Answer (A)

Sol. $4x - 3y = 12\alpha,$

$$4x + 3y = \frac{12}{\alpha}$$

$\Rightarrow \quad (4x)^2 - (3y)^2 = 12^2$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{4^2} = 1$$

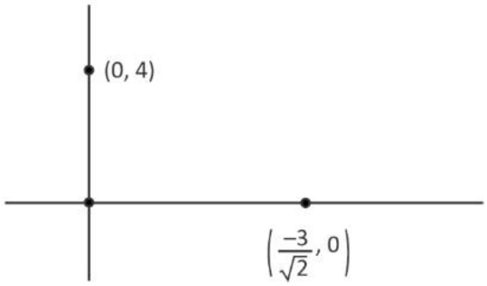
$y = mx \pm \sqrt{a^2m^2 - b^2}$ for hyperbola

$m = \frac{4\sqrt{2}}{3}$ (as parallel to $4x - \frac{3}{\sqrt{2}}y = 0$)

$$y = \frac{4\sqrt{2}x}{3} \pm \sqrt{9 \times \frac{32}{9} - 16} = \frac{4\sqrt{2}x}{3} \pm 4$$

$y = \frac{4\sqrt{2}x}{3} + 4$, as $(0, q)$ $q > 0$

$p = \frac{-3}{\sqrt{2}}, q = 4$



$$\Rightarrow pq = \frac{-12}{\sqrt{2}} = \frac{-12\sqrt{2}}{2} = -6\sqrt{2}$$

SECTION 2 (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	−2	In all other cases.

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5. Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$.

Then which of the following statements is (are) TRUE?

(A) The determinant of $Q - 2I$ is zero

(B) The determinant of $Q - 6I$ is 12

(C) The determinant of $Q - 3I$ is 15

(D) $yz = 2$

Answer (A, B)

Sol. $P=\begin{pmatrix}2&0\\0&3\end{pmatrix}$, $Q=\begin{pmatrix}x&y\\z&4\end{pmatrix}$ let $R=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $x, y, z, a, b, c, d \neq 0$

Now, $QR = RP$

$\Rightarrow \quad |Q||R|=|R||P| \Rightarrow \boxed{|R|=0 \text{ or } |P|=|Q|}$

$QR=RP \Rightarrow \begin{pmatrix}ax+cy & bx+dy\\ az+4c & bz+4d\end{pmatrix}=\begin{pmatrix}2a & 3b\\ 2c & 3d\end{pmatrix}$

Comparing $az = -2c$ and $bz = -d$

$\Rightarrow \quad \frac{a}{b}=\frac{2c}{d}$

$ad - bc = bc \neq 0$

$\because \quad b \neq 0$ and $c \neq 0$

$\Rightarrow \quad |R| \neq 0$

$\Rightarrow \quad |P|=|Q| \Rightarrow \boxed{4x-yz=6}$ ...(i)

Now $ax + cy = 2a$ and $az + 4c = 2c$

$\left.\begin{array}{l} a(x-z)+c\cdot y=0\\ az\ +2c=0\end{array}\right]$ this system has non-trivial sol as a and c are non zero.

$\Rightarrow \quad \begin{vmatrix}x-2 & y\\ z & 2\end{vmatrix}=0 \Rightarrow \boxed{2x-yz=4}$ ...(ii)

Solving (i) and (ii)

$x = 1$ and $yz = -2$

Now $|Q-2I|=\begin{vmatrix}x-2 & y\\ z & 2\end{vmatrix}=2x-4-yz=0$ (A)

$|Q-3I|=\begin{vmatrix}x-3 & y\\ z & 1\end{vmatrix}=x-3-yz=0$

$|Q-6I|=\begin{vmatrix}x-6 & y\\ z & -2\end{vmatrix}=-2x+12-yz=12$ (B)

6. Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let $\mathcal{R}$ denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(A) $(4, \sqrt{3}) \in S$

(B) $(5, \sqrt{2}) \in S$

(C) Area of R is $\frac{14}{3}-2\sqrt{3}$

(D) Area of R is $\frac{14}{3}-\sqrt{3}$

Answer (A, C)

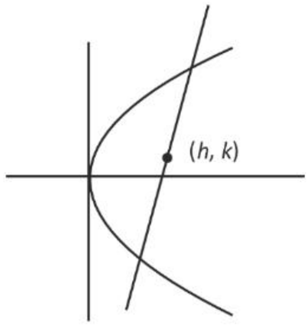
Sol. $y^2 = x$

chord with given mid point

$T = S_1$

$$yk - \frac{x+h}{2} = k^2 - h$$

$$2ky - x - h = 2k^2 - 2h$$



Now,

$$A = \int_{y_1}^{y_2} (2ky - 2k^2 + h) - y^2)dy = \frac{4}{3}$$

$$\left(ky^2 + (h-2k^2)y - \frac{y^3}{3}\right)_{y_1}^{y_2} = \frac{4}{3}$$

$$k(y_2^2 - y_1^2) + (h-2k^2)(y_2 - y_1) - \frac{1}{3}(y_2^3 - y_1^3) = \frac{4}{3}$$

$$(y_2 - y_1)\left[k - 2k + h - 2k^2 - \frac{1}{3}(4k^2 - 2k^2 + h)\right] = \frac{4}{3}$$

$$2(h-k^2)^{1/2}[2h - 2k^2] = 4$$

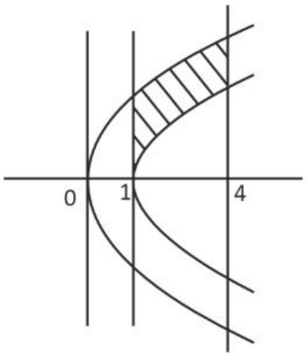
$$(h-k^2)^{3/2} = 1$$

$$x - y^2 = 1 \Rightarrow y^2 = x - 1$$

$$A = \int_1^4 (\sqrt{x} - \sqrt{x-1})dx$$

$$= \frac{2}{3}\left[x^{3/2} - (x-1)^{3/2}\right]_1^4 = \frac{2}{3}\left[8 - 3\sqrt{3} - 1\right]$$

$$= \frac{14}{3} - 2\sqrt{3}$$



Solving chord and parabola

$$yk - \frac{y+h}{2} = k^2 - h$$

$$2ky - y^2 - h = 2k^2 - 2h$$

$$y^2 - 2ky + 2k^2 - h = 0$$

$$y_1 + y_2 = 2k$$

$$y_1 y_2 = 2k^2 - h$$

$$y_2 - y_1 = \sqrt{4k^2 - 8k^2 + 4x}$$

$$= 2\sqrt{h - k^2}$$

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point (3, 0). Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where O denotes the origin (0, 0). Let $|XY|$ denote the length of the line segment XY.

Then which of the following statements is (are) TRUE?

(A) The equation of the line joining P and Q is $2x + 3y = 3\ (1 + \sqrt{3})$

(B) The equation of the line joining P and Q is $2x + y = 3\ (1 + \sqrt{3})$

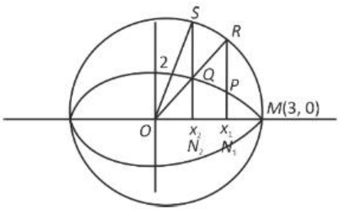
(C) If $N_2 = (x_2, 0)$, then $3\ |N_2Q| = 2\ |N_2S|$

(D) If $N_1 = (x_1, 0)$, then $9\ |N_1P| = 4\ |N_1R|$

Answer (A, C)

Sol. $P\left(3\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6}\right) = P\left(\frac{3\sqrt{3}}{2}, 1\right)$

$Q\left(3\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3}\right) = Q\left(\frac{3}{2}, \sqrt{3}\right)$



Line PQ:

$$y-1=\frac{\sqrt{3}-1}{\frac{3}{2}-\frac{3\sqrt{3}}{2}}\left(x-\frac{3\sqrt{3}}{2}\right)$$

$$y-1=\frac{-2}{3}\left(x-\frac{3\sqrt{3}}{2}\right)$$

$$3y-3=-2x+3\sqrt{3}$$

$$2x+3y=3(1+\sqrt{3})$$ (A)

Now, $\frac{N_2Q}{N_2S}=\frac{b}{a}=\frac{2}{3}$ (C)

And $\frac{N_1P}{N_1R}=\frac{b}{a}=\frac{2}{3}$

8. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}\frac{6x+\sin x}{2x+\sin x} & \text{if } x\neq 0,\\ \frac{7}{3} & \text{if } x=0.\end{cases}$$

Then which of the following statements is (are) TRUE?

(A) The point $x = 0$ is a point of local maxima of f

(B) The point $x = 0$ is a point of local minima of f

(C) Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3

(D) Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

Answer (B, C, D)

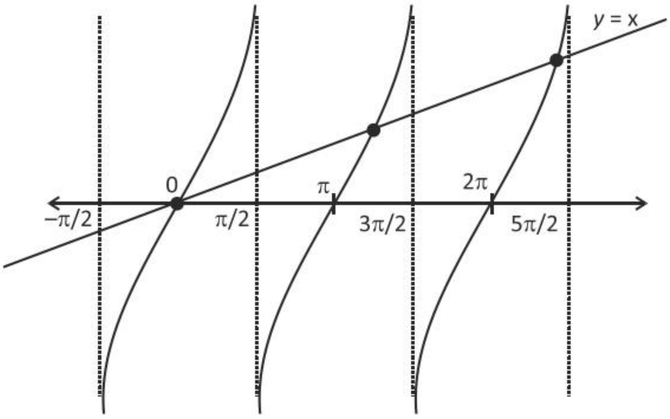
Sol. $f(x)$ is continuous but not differentiable at $x = 0$

However, $f'(x)=\frac{4(\sin x-x\cos x)}{(2x+\sin x)^2}=\frac{4\cos x(\tan x-x)}{(2x+\sin x)^2}$

For $x\neq 0$

$f'(0^-)<0$ and $f'(0^+)>0$

$\therefore$ $f(x)$ has a local minimum at $x = 0$



Considering the sign change of $f'(x)$ at darkened points, its clear that.

There is one point of local maximum in each of the intervals $\left(\pi, \frac{3\pi}{2}\right), \left(3\pi, \frac{7\pi}{2}\right), \left(5\pi, \frac{11\pi}{2}\right)$.

And one point of local minimum in each of the intervals $\left(2\pi, \frac{5\pi}{2}\right), \left(4\pi, \frac{9\pi}{2}\right)$.

SECTION 3 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	If **ONLY** the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

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9. Let $y(x)$ be the solution of the differential equation

$$x^2\frac{dy}{dx} + xy = x^2 + y^2,\ x > \frac{1}{e},$$

satisfying $y(1) = 0$. Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is ___________.

Answer (00.75)

Sol. $\frac{dy}{dx} + \frac{y}{x} = \frac{x^2}{x^2} + \frac{y^2}{x^2}$

$$\frac{dy}{dx}+\frac{y}{x}=1+\left(\frac{y}{x}\right)^2$$

Let $\frac{y}{x}=t$

$y = xt$

$$\frac{dy}{dx}=x\frac{dt}{dx}+t$$

$$\therefore\ x\frac{dt}{dx}+t+t=1+t^2$$

$$\Rightarrow x\frac{dt}{dx}+2t=1+t^2$$

$$\Rightarrow x\frac{dt}{dx}+t^2+1-2t$$

$$\Rightarrow x\frac{dt}{dx}=(t-1)^2$$

$$\Rightarrow \frac{dt}{(t-1)^2}=\frac{dx}{x}$$

Integrating both sides

$$\Rightarrow \int\frac{dt}{(t-1)^2}=\int\frac{dx}{x}$$

$$\Rightarrow \frac{-1}{(t-1)}=\ln x+C$$

$$\Rightarrow \frac{-1}{\frac{y}{x}-1}=\ln x+C$$

$$\Rightarrow \frac{-x}{y-x}=\ln x+C$$

Given $y(1) = 0$

$1 = C$

$$\Rightarrow \frac{-x}{y-x}=\ln x+1 \qquad ...(i)$$

$\therefore$ Put $x = e$

$$\frac{-e}{y-e}=1+1$$

$$\Rightarrow -e=2(y-e)$$

$$\Rightarrow e=2(e-y)$$

$$\Rightarrow \frac{e}{2}=e-y$$

$$\Rightarrow y=e-\frac{e}{2} \Rightarrow \frac{e}{2} \Rightarrow \boxed{y=\frac{e}{2}}$$

Put $x = e^2$ in (i)

$$\Rightarrow \frac{-e^2}{y-e^2}=2+1$$

$$\Rightarrow -e^2=3(y-e^2)$$

$$\Rightarrow -e^2=3y-3e^2$$

$$\Rightarrow 2e^2=3y$$

$$\Rightarrow y=\frac{2}{3}e^2$$

$$\therefore \frac{2(y(e))^2}{y(e^2)}=2\frac{\left(\frac{e}{2}\right)^2}{\frac{2}{3}e^2}$$

$$\Rightarrow \frac{\frac{1}{2}}{\frac{2}{3}}=\frac{3}{4}=00.75$$

10. Let $a_0, a_1, \ldots, a_{23}$ be real numbers such that

$$\left(1+\frac{2}{5}x\right)^{23}=\sum_{i=0}^{23}a_i x^i$$

For every real number x. Let a_r be the largest among the numbers a_j for $0 \le j \le 23$.

Then the value of r is ___________.

Answer (06.00)

Sol. For $\left(1+\frac{2x}{5}\right)^{23}$

The numerically greatest term is $\frac{(n+1)}{1+\left|\frac{5}{2x}\right|}$

$\therefore$ Put $x = 1$, $n = 23$

$\therefore \frac{n+1}{1+\left|\frac{5}{2x}\right|}=\frac{24}{1+\frac{5}{2}}=\frac{48}{7}=6+f(0<f<1)$

$\therefore$ T_7 is numerically greatest term

$\therefore$ a_6 is largest.

11. A factory has a total of three manufacturing units, M_1, M_2, and M_3, which produce bulbs independent of each other. The units M_1, M_2, and M_3 produce bulbs in the proportions of 2 : 2 : 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is ____________.

Answer (00.30)

Sol. H_1 : bulb is produced by unit M_1.

H_2 : bulb is produced by unit M_2.

H_3 : bulb is produced by unit M_3.

E : bulb produced in defective

$P(H_1)=\frac{2}{5}, P(H_2)=\frac{2}{5} P(H_3)=\frac{1}{5}$

$P(E/H_1)=\frac{15}{100}=\frac{3}{20}$

$P(E)=P(E/H_1)\cdot P(H_1)+P(E/H_2)\cdot P(H_2)+P(E/H_3)\cdot P(H_3)$

$\Rightarrow \frac{20}{100}=\frac{3}{20}.\frac{2}{5}+P(E/H_2)\cdot\frac{2}{5}+P(E/H_3)\frac{1}{5}$

$\Rightarrow 2P(E/H_2)+P(E/H_3)=1-\frac{3}{10}=\frac{7}{10}$ ...(i)

Now given $P(H_2/E)=\frac{P(E/H_2)P(H_2)}{P(E)}$

$$\frac{2}{5}=\frac{P(E/H_2)\frac{2}{5}}{\frac{1}{5}}$$

$$\Rightarrow P(E/H_2)=\frac{1}{5} \qquad \text{...(ii)}$$

from (i) and (ii)

$$P(E/H_3)=\frac{3}{10}$$

Required probability = $P(E/H_3)$ = 0.30

12. Consider the vectors

$$\vec{x}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{y}=2\hat{i}+3\hat{j}+\hat{k} \text{ and } \vec{z}=3\hat{i}+\hat{j}+2\hat{k}.$$

For two distinct positive real numbers α and β, define

$$\vec{X}=\alpha\vec{x}+\beta\vec{y}-\vec{z},\ \vec{Y}=\alpha\vec{y}+\beta\vec{z}-\vec{x}, \text{ and } \vec{Z}=\alpha\vec{z}+\beta\vec{x}-\vec{y}.$$

If the vectors $\vec{X},\vec{Y}$, and $\vec{Z}$ lie in a plane, then the value of $\alpha+\beta-3$ is __________

Answer (–2.00)

Sol. $[\vec{X}\vec{Y}\vec{Z}]=0$ since X, Y, Z lie in plane

$$\Rightarrow [\vec{x}\vec{y}\vec{z}]\begin{bmatrix}\alpha & \beta & -1\\ -1 & \alpha & \beta\\ \beta & -1 & \alpha\end{bmatrix}=0$$

$$[\vec{x}\vec{y}\vec{z}]\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ -3 & 1 & 2\end{bmatrix}=-18$$

$$\Rightarrow \begin{bmatrix}\alpha & \beta & -1\\ -1 & \alpha & \beta\\ \beta & -1 & \alpha\end{bmatrix}=0$$

$$\Rightarrow \alpha^3+\beta^3+3\alpha\beta-1=0$$

$$\Rightarrow \alpha^3+\beta^3+(-1)^3=3(-1)(\alpha)(\beta)$$

$\Rightarrow \alpha + \beta - 1 = 0$ or $\alpha = \beta = -1$

but $\alpha \neq \beta$ and $\alpha, \beta > 0$

$\Rightarrow \alpha + \beta = 1 \Rightarrow \alpha + \beta - 3 = -2$

13. For a non-zero complex number z, let arg(z) denote the principal argument of z, with $-\pi < \arg(z) \leq \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$$

Then the value of $\frac{3\alpha}{\pi}$ is ______________.

Answer (–2.00)

Sol. $\alpha = \arg\left(\sum_{n-1}^{2025} (-\omega)^n\right)$

$= \arg(-\omega + \omega^2 - \omega^3 + \omega^4 - \omega^2 + \omega^6 + \ldots\ldots)$

(sum of 6 consecutive terms are zero)

$= \arg(-\omega + \omega^2 - \omega^3)$

$= \arg(2\omega^2)$

$= -\frac{2\pi}{3}$

$\therefore \quad \frac{3\alpha}{\pi} = -2$

14. L et $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to (0, 4)$ be functions defined by

$f(x) = \log_e(x^2 + 2x + 4)$, and $g(x) = \frac{4}{1+e^{-2x}}$. Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f(g^{-1}(x))$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is ______________.

Answer (00.25)

Sol. $f : \mathbb{R} \to \mathbb{R}$

$f(x) = \log_e(x^2 + 2x + 4) = \log_e[(x + 1)^2 + 3]$

$g : \mathbb{R} \to (0, 4)$

$$g(x) = \frac{4}{1+e^{-2x}}$$

$$y = \frac{4}{1+e^{-2x}} \Rightarrow 1+ e^{-2x} = \frac{4}{y}$$

$$\Rightarrow e^{-2x} = \frac{4}{y} - 1$$

$$\Rightarrow -2x = \ln\left(\frac{4}{y} - 1\right)$$

$$\Rightarrow x = -\frac{1}{2}\ln\left(\frac{4-y}{y}\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{y}{4-y}\right)$$

$$\Rightarrow \boxed{g^{-1}(x) = \frac{1}{2}\ln\left(\frac{x}{4-x}\right)} \quad g^{-1}(2) = 0$$

$$\therefore f(g^{-1}(x)) = \log_e[(g^{-1}(x) + 1)^2 + 3]$$

$$\frac{d}{dx}(f(g^{-1}(x)) = \frac{1}{[(g^{-1}(x)+1)^2+3]}(2)(g^{-1}(x)+1) \times \frac{d}{dx}(g^{-1}(x))$$

$$\frac{d}{dx}(f(g^{-1}(x)) = \frac{2(g^{-1}(x)+1)}{(g^{-1}(x)+1)^2+3} \times \frac{d}{dx}\left(\frac{1}{2}\ln\left(\frac{x}{4-x}\right)\right)$$

$$= \frac{2(g^{-1}(x)+1)}{(g^{-1}(x)+1)^2+3} \times \frac{1}{2} \times \frac{4-x}{x} \times \frac{4-x+x}{(4-x)^2}$$

$$\left.\frac{d}{dx} f(g^{-1}(x)\right|_{x=2} = \frac{2(g^{-1}(2)+1)}{(g^{-1}(2)+1)^2+3} \times \frac{1}{2} \times \frac{2}{2} \times \frac{4}{4}$$

$$= \frac{2}{1+2} \times \frac{1}{3} \quad \text{as } g^{-1}(2) = 0$$

$$= \frac{1}{4}$$

$$= 0.25$$

15. Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \ldots + \frac{1}{\sin 118^\circ \sin 119^\circ}.$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2$$

is ______________.

Answer (03.00)

Sol. $\alpha = \dfrac{1}{\sin 60^\circ \cdot \sin 61^\circ} + \dfrac{1}{\sin 62^\circ \cdot \sin 63^\circ} + \ldots + \dfrac{1}{\sin 118^\circ \cdot \sin 119^\circ}$

$$\sin 1^\circ \cdot \alpha = \frac{\sin(61^\circ - 60^\circ)}{\sin 60^\circ \cdot \sin 61^\circ} + \frac{\sin(63^\circ - 62^\circ)}{\sin 62^\circ \cdot \sin 63^\circ} + \ldots + \frac{\sin(119^\circ - 118^\circ)}{\sin 118^\circ \cdot \sin 119^\circ}$$

$$\sin 1^\circ \cdot \alpha = \cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63^\circ + \ldots \cot 118^\circ - \cot 119^\circ$$

$= \cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63^\circ + \ldots + \cot 89^\circ + \cot 90^\circ + \cot 89^\circ - \ldots\ldots - \cot 62^\circ + \cot 61^\circ$

$= \cot 60^\circ$

$$\alpha = \frac{1}{\sqrt{3}\sin 1^\circ} = \frac{\operatorname{cosec} 1^\circ}{\sqrt{3}}$$

$$\therefore \left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2 = 3.$$

16. If

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx,$$

then the value of $\sqrt{7}\tan\left(\dfrac{2\alpha\sqrt{7}}{\pi}\right)$ is _______.

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.)

Answer (21.00)

Sol. $\alpha = \displaystyle\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx$ ...(1) Put $x \to \dfrac{1}{x}$

$$= -\int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\frac{1}{x}}{\frac{2x^2-3x+2}{x^2}} \times \frac{1}{x^2}dx \qquad dx \to -\frac{1}{x^2}dx$$

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\cot^{-1}x}{2x^2-3x+2}dx \qquad ...(2) \qquad \because \tan^{-1}\frac{1}{x} = \cot^{-1}x\,(x>0)$$

Add (1) and (2)

$$2\alpha = \int_{\frac{1}{2}}^{2} \frac{\frac{\pi}{2}}{2x^2-3x+2}dx \qquad \because \cot^{-1}x + \tan^{-1}x = \frac{\pi}{2}$$

$$2\alpha = \frac{\pi}{4}\int_{\frac{1}{2}}^{2} \frac{dx}{x^2-\frac{3}{2}x+1} = \frac{\pi}{4}\int_{\frac{1}{2}}^{2} \frac{dx}{\left(x-\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$2\alpha = \frac{\pi}{4} \times \frac{4}{\sqrt{7}}\left[\tan^{-1}\left(\frac{4x-3}{\sqrt{7}}\right)\right]_{1/2}^{2} \qquad \because \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\frac{2\alpha\sqrt{7}}{\pi} = \tan^{-1}\frac{5}{\sqrt{7}} - \tan^{-1}\left(-\frac{1}{\sqrt{7}}\right)$$

$$\frac{2\alpha\sqrt{7}}{\pi} = \tan^{-1}\frac{5}{\sqrt{7}} + \tan^{-1}\frac{1}{\sqrt{7}}$$

$$\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = \frac{\frac{5}{7}+\frac{1}{\sqrt{7}}}{1-\frac{5}{7}} = 3\sqrt{7}$$

$$\Rightarrow \sqrt{7}\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = 21$$

PART-II : PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. A temperature difference can generate e.m.f. in some materials. Let S be the e.m.f. produced per unit temperature difference between the ends of a wire, σ the electrical conductivity and κ the thermal conductivity of the material of the wire. Taking M, L, T, I and K as dimensions of mass, length, time, current and temperature, respectively, the dimensional formula of the quantity $Z = \frac{S^2\sigma}{\kappa}$ is

(A) $[M^0L^0T^0I^0K^0]$

(B) $[M^0L^0T^0I^0K^{-1}]$

(C) $[M^1L^2T^{-2}I^{-1}K^{-1}]$

(D) $[M^1L^2T^{-4}I^{-1}K^{-1}]$

Answer (B)

Sol. According to Seeback effect

$$(ZT) = \frac{S^2\sigma T}{K} = \text{dimensionless}$$

$$Z \equiv \frac{1}{T}$$

$[Z] = [K^{-1}]$

2. Two co-axial conducting cylinders of same length ℓ with radii $\sqrt{2}R$ and $2R$ are kept, as shown in Fig. 1. The charge on the inner cylinder is Q and the outer cylinder is grounded. The annular region between the cylinders is filled with a material of dielectric constant $\kappa = 5$. Consider an imaginary plane of the same length ℓ at a distance R from the common axis of the cylinders. This plane is parallel to the axis of the cylinders. The cross-sectional view of this arrangement is shown in Fig. 2. Ignoring edge effects, the flux of the electric field through the plane is (ε_0 is the permittivity of free space):

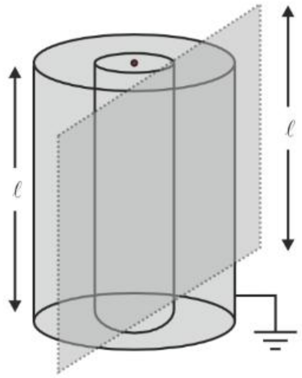


Fig. 1

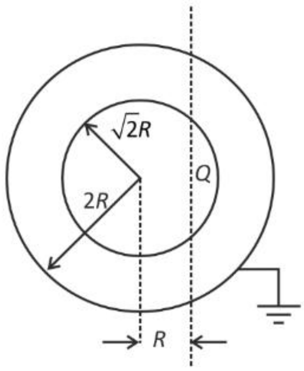


Fig. 2

(A) $\frac{Q}{30\varepsilon_0}$

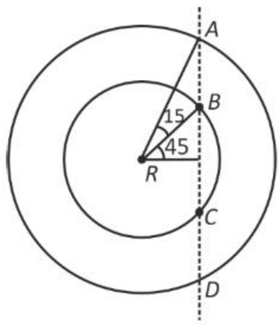
(B) $\frac{Q}{15\varepsilon_0}$

(C) $\frac{Q}{60\varepsilon_0}$

(D) $\frac{Q}{120\varepsilon_0}$

Answer (C)

Sol.



$\phi_{BC} = 0$

$\phi_{AB} = \phi_{CD}$

$$\phi_{AB} = \frac{15}{360} \times \frac{Q}{K\varepsilon_0}$$

$$= \frac{Q}{24K\varepsilon_0}$$

$$\phi_{net} = 2\phi_{AB} = \frac{Q}{12K\varepsilon_0}$$

$$= \frac{Q}{60\varepsilon_0}$$

3. As shown in the figures, a uniform rod OO' of length l is hinged at the point O and held in place vertically between two walls using two massless springs of same spring constant. The springs are connected at the midpoint and at the top-end (O') of the rod, as shown in Fig. 1 and the rod is made to oscillate by a small angular displacement. The frequency of oscillation of the rod is f_1. On the other hand, if both the springs are connected at the midpoint of the rod, as shown in Fig. 2 and the rod is made to oscillate by a small angular displacement, then the frequency of oscillation is f_2. Ignoring gravity and assuming motion only in the plane of the diagram, the value of $\frac{f_1}{f_2}$ is

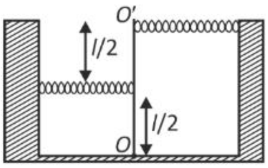


Fig. 1

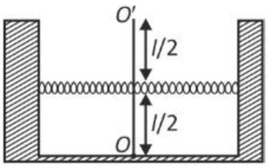


Fig. 2

(A) 2

(B) $\sqrt{2}$

(C) $\sqrt{\frac{5}{2}}$

(D) $\sqrt{\frac{2}{5}}$

Answer (C)

Sol.

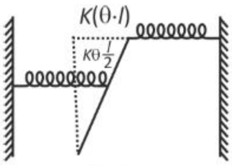


Fig. 1

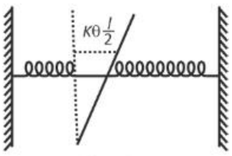


Fig. 2

$$\tau_{net} = K(\theta \cdot l) \cdot l + K\left(\theta \cdot \frac{l}{2}\right)\frac{l}{2} \qquad \tau_{net} = \left(K\theta\frac{l}{2}\right)\frac{l}{2} \times 2$$

$$= \frac{5Kl^2}{4}\theta = I\alpha \qquad = \frac{Kl^2}{2}\theta = I\alpha$$

as $\alpha = \omega^2\theta$

$$\Rightarrow \frac{\omega_1^2}{\omega_2^2} = \frac{\frac{5Kl^2}{4Kl^2}}{2} = \frac{5}{2}$$

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{5}{2}}$$

4. Consider a star of mass m_2 kg revolving in a circular orbit around another star of mass m_1 kg with $m_1 \gg m_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. In this transfer process, there is no other loss of mass. If the separation between the centers of the stars is r, then its relative rate of change $\frac{1}{r}\frac{dr}{dt}$ (in s^{-1}) is given by:

(A) $-\frac{3\gamma}{2m_2}$

(B) $-\frac{2\gamma}{m_2}$

(C) $-\frac{2\gamma}{m_1}$

(D) $-\frac{3\gamma}{2m_1}$

Answer (B*)

Sol. $\frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r} = m_2\omega^2 r \frac{m_2}{m_2}\frac{r^3}{r^3}$

$\frac{Gm_1m_2}{r^2} = \frac{L^2}{m_2r^3}$

$\frac{G}{L^2} = \frac{1}{m_1m_2^2r}$

$m_1m_2^2r = \text{constant}$

$\frac{dm_1}{m_1} + \frac{2dm_2}{m_2} + \frac{dr}{r} = 0$

$$\boxed{\begin{array}{l} \frac{dm_1}{dt} = +\lambda \\ \frac{dm_2}{dt} = -\lambda \end{array}}$$

$\frac{dm_1}{dt\, m_1} + \frac{2dm_2}{dt\, m_2} + \frac{dr}{dtr}$

$\frac{\lambda}{m_1} + \frac{(-2)\lambda}{m_2} + \frac{dr}{dtr} = 0$

$m_1 >> m_2$

$\frac{dr}{rdt} = \frac{2\lambda}{m_2}$

SECTION 2 (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+ 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

5. A positive point charge of 10^{-8} C is kept at a distance of 20 cm from the center of a neutral conducting sphere of radius 10 cm. The sphere is then grounded and the charge on the sphere is measured. The grounding is then removed and subsequently the point charge is moved by a distance of 10 cm further away from the center of the sphere along the radial direction. Taking $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/C^2$ (where ε_0 is the permittivity of free space), which of the following statements is/are correct?

(A) Before the grounding, the electrostatic potential of the sphere is 450 V.

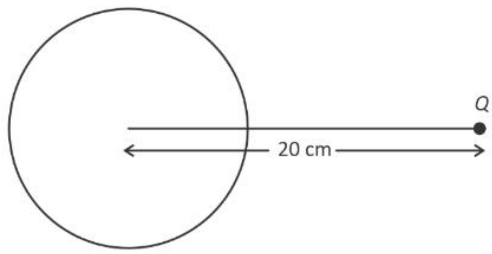
(B) Charge flowing from the sphere to the ground because of grounding is 5×10^{-9} C.

(C) After the grounding is removed, the charge on the sphere is -5×10^{-9} C.

(D) The final electrostatic potential of the sphere is 300 V.

Answer (A, B, C)

Sol.



(A) $V = \frac{KQ}{0.2} = \frac{9\times10^9\times10^{-8}}{0.2} = 90\times5 = 450\text{ V}$

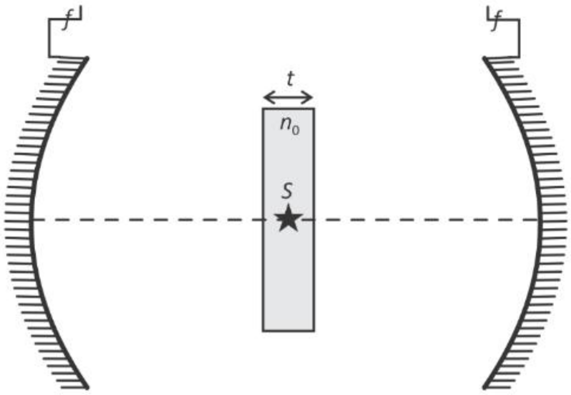
(B) $\frac{KQ}{0.2} = \frac{Kq}{0.1} \Rightarrow q = \frac{Q}{2} = \frac{10^{-8}}{2} = 5\times10^{-9}\text{C}$

(D) $\frac{KQ}{0.3} + \frac{K\left(\frac{-Q}{2}\right)}{0.1} = KQ\left(\frac{1}{0.3} - \frac{3}{0.3}\right)$

$\Rightarrow -2\left(\frac{KQ}{0.3}\right) = \frac{-2\times9\times10^9\times10^{-8}}{3\times10^{-1}}$

$= -600\text{ V}$

6. Two identical concave mirrors each of focal length f are facing each other as shown in the schematic diagram. The focal length f is much larger than the size of the mirrors. A glass slab of thickness t and refractive index n_0 is kept equidistant from the mirrors and perpendicular to their common principal axis. A monochromatic point light source S is embedded at the center of the slab on the principal axis, as shown in the schematic diagram. For the image to be formed on S itself, which of the following distances between the two mirrors is/are correct?



(A) $4f + \left(1 - \frac{1}{n_0}\right)t$

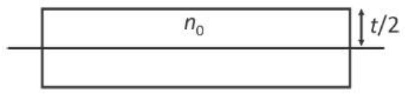
(B) $2f + \left(1 - \frac{1}{n_0}\right)t$

(C) $4f + (n_0 - 1)t$

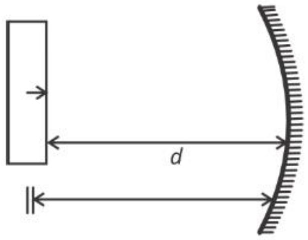
(D) $2f + (n_0 - 1)t$

Answer (A, B)

Sol.



$$h' = \frac{h_0}{\mu_0}$$



$$\Rightarrow \quad d + \frac{t}{2n_0} = 2f$$

$$d = 2f - \frac{t}{2n_0}$$

$$d + \frac{t}{2} = 2f - \frac{t}{2n_0} + \frac{t}{2}$$

$$2\left(d + \frac{t}{2}\right) = 4f + \frac{t}{2}\left(1 - \frac{1}{n_0}\right)$$

$$= 4f + t\left(1 - \frac{1}{n_0}\right)$$

$$d + \frac{t}{2n_0} = f$$

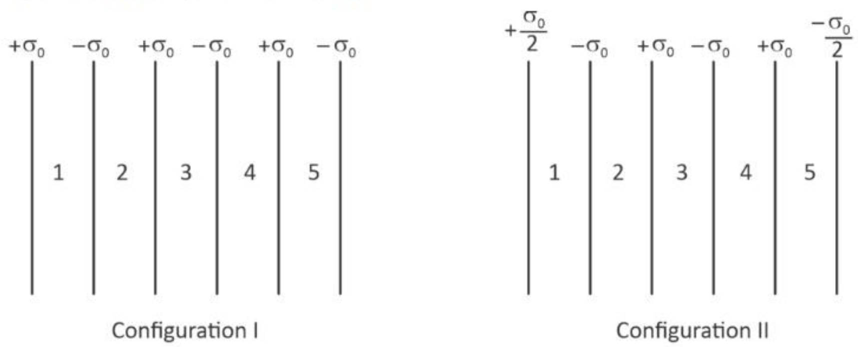
$$d = f - \frac{t}{2n_0}$$

$$d + \frac{t}{2} = f - \frac{t}{2n_0} + \frac{t}{2}$$

$$2\left(d + \frac{t}{2}\right) = 2f + t\left(1 - \frac{1}{n_0}\right)$$

7. Six infinitely large and thin non-conducting sheets are fixed in configurations I and II. As shown in the figure, the sheets carry uniform surface charge densities which are indicated in terms of σ_0. The separation between any two consecutive sheets is 1 μm. The various regions between the sheets are denoted as 1, 2, 3, 4 and 5. If $\sigma_0 = 9\ \mu C/m^2$, then which of the following statements is/are correct?

(Take permittivity of free space $\varepsilon_0 = 9 \times 10^{-12}$ F/m)



(A) In region 4 of the configuration I, the magnitude of the electric field is zero.

(B) In region 3 of the configuration II, the magnitude of the electric field is $\frac{\sigma_0}{\varepsilon_0}$.

(C) Potential difference between the first and the last sheets of the configuration I is 5 V.

(D) Potential difference between the first and the last sheets of the configuration II is zero.

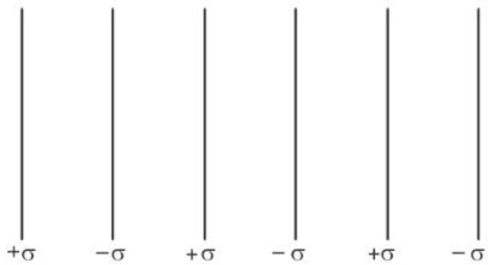
Answer (A)

Sol. In I region 4, $E = 0$ As total charge on left = Total charge on right.

In II region 3,

$$E = \frac{\sigma}{4\varepsilon_0} + \frac{\sigma}{4\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

In I, ΔV



$$\Delta V = \frac{3Q}{C} = \frac{3Qd}{A\varepsilon_0} = \frac{3\sigma d}{\varepsilon_0} = 3\text{ V}$$

In II,

$$\Delta V = \frac{Q}{2C} = \frac{Qd}{2A\varepsilon_0} = \frac{\sigma d}{2\varepsilon_0}$$

8. The efficiency of a Carnot engine operating with a hot reservoir kept at a temperature of 1000 K is 0.4. It extracts 150 J of heat per cycle from the hot reservoir. The work extracted from this engine is being fully used to run a heat pump which has a coefficient of performance 10. The hot reservoir of the heat pump is at a temperature of 300 K. Which of the following statements is/are correct?

(A) Work extracted from the Carnot engine in one cycle is 60 J.

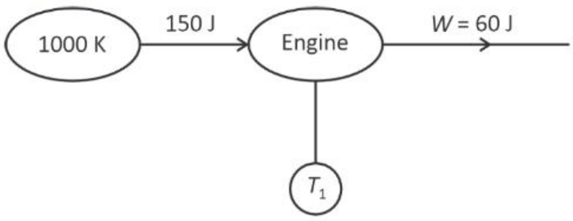
(B) Temperature of the cold reservoir of the Carnot engine is 600 K.

(C) Temperature of the cold reservoir of the heat pump is 270 K.

(D) Heat supplied to the hot reservoir of the heat pump in one cycle is 540 J.

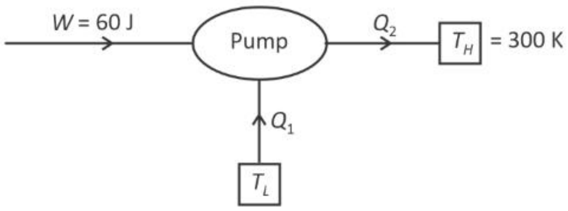
Answer (A, B, C)

Sol.



$$1 - \frac{T_1}{1000} = 0.4$$

$T_1 = 600$ K



$$\frac{Q_1}{60} = 10$$

$Q_1 = 600$

$Q_2 = 660$

$$\eta = \frac{1}{\text{Cop}} = 0.1$$

$$1 - \frac{T_L}{300} = 0.1$$

$$T_L = 270 \text{ K}$$

SECTION 3 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+4	If ONLY the correct numerical value is entered in the designated place;
Zero Marks	:	0	In all other cases.

---

9. A conducting solid sphere of radius R and mass M carries a charge Q. The sphere is rotating about an axis passing through its center with a uniform angular speed ω. The ratio of the magnitudes of the magnetic dipole moment to the angular momentum about the same axis is given as $\alpha \frac{Q}{2M}$. The value of α is ______

Answer (01.67)

Sol. $$\frac{M}{L} = \frac{q}{2\text{ m}}$$

$$M_{\text{shell}} = M_{\text{conducting shell}}$$

$$M_{CS} = \left(\frac{q}{2\text{ m}}\right)\frac{2}{3}mr^2\omega$$

$$L_{SS} = \frac{2}{5}mr^2\omega$$

$$\frac{M}{L} = \frac{q}{2\text{ m}}\,\frac{2}{3} \times \frac{5}{2}$$

$$= \left(\frac{q}{2\text{ m}}\right)\frac{5}{3}$$

$$\boxed{\alpha = 01.67}$$

10. A hydrogen atom, initially at rest in its ground state, absorbs a photon of frequency ν_1 and ejects the electron with a kinetic energy of 10 eV. The electron then combines with a positron at rest to form a positronium atom in its ground state and simultaneously emits a photon of frequency ν_2. The center of mass of the resulting positronium atom moves with a kinetic energy of 5 eV. It is given that positron has the same mass as that of electron and the positronium atom can be considered as a Bohr atom, in which the electron and the positron orbit around their center of mass. Considering no other energy loss during the whole process, the difference between the two photon energies (in eV) is ______

Answer (11.80)

Sol. $E_1 = \mathrm{E}_{\text{ionisation}} + \mathrm{KE}_e$

$= 13.6 + 10 = 23.6$ eV

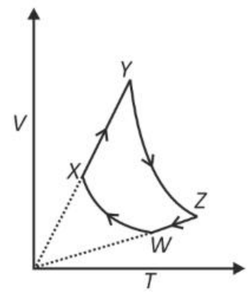
$10 \text{ eV} = (\mathrm{KE})_{\text{com}} +$ Ground State energy + $h\nu_2$

$10 \text{ eV} = 5 \text{ eV} - 6.8 \text{ eV} + h\nu_2$

$E_2 = h\nu_2 = 11.8$ eV

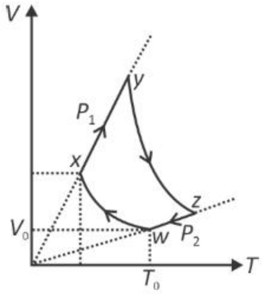
$E_1 - E_2 = 11.80$ eV

11. An ideal monatomic gas of n moles is taken through a cycle $WXYZW$ consisting of consecutive adiabatic and isobaric quasi-static processes, as shown in the schematic V-T diagram. The volume of the gas at W, X and Y points are, 64 cm^3, 125 cm^3 and 250 cm^3, respectively. If the absolute temperature of the gas T_W at the point W is such that nRT_W = 1 J (R is the universal gas constant), then the amount of heat absorbed (in J) by the gas along the path XY is ______



Answer (01.60)

Sol.



$$Q_{xy} = P_1(V_y - V_x) + nR(T_y - T_x)\frac{3}{2}$$

$$Q_{xy} = \frac{5}{2}R(T_y - T_x)n$$

$$T_w \cdot (64)^{\frac{2}{3}} = T_x(125)^{\frac{2}{3}}$$

$$T_w \cdot 16 = T_x \cdot 25$$

$$T_x = \frac{16}{25}T_w$$

$$\frac{T_y}{V_y} = \frac{T_x}{V_x}$$

$$T_y = T_x\left(\frac{250}{125}\right) = 2T_x$$

$$Q_{xy} = \frac{5}{2}nR(T_x)$$

$$= \frac{5}{2}nR \cdot \frac{16}{25}T_w$$

$$= \frac{8}{5}$$

$$Q_{xy} = 1.6$$

12. A geostationary satellite above the equator is orbiting around the earth at a fixed distance r_1 from the center of the earth. A second satellite is orbiting in the equatorial plane in the opposite direction to the earth's rotation, at a distance r_2 from the center of the earth, such that $r_1 = 1.21\, r_2$. The time period of the second satellite as measured from the geostationary satellite is $\frac{24}{p}$ hours. The value of p is ________

Answer (02.33)

Sol. $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

$\frac{T_1^2}{T_2^2} = (1.21)^3$

r_1 = geostationary

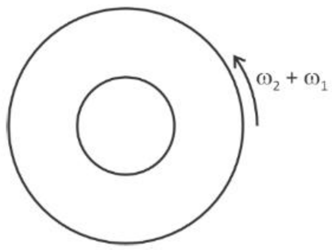
$r_2 = \frac{r_1}{1.21}$

$\frac{T_1^2}{T_2^2} = (1.1)^6$

$\frac{T_1}{T_2} = (1.1)^3$

$\frac{T_1}{T_2} = 1.331$

$\omega_2 = (1.331)\omega_1$

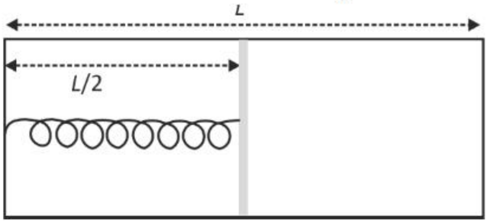


$T_{measured} = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{2.331\omega_1}$

$= \frac{24}{2.331} = \frac{24 \times 3}{7}$

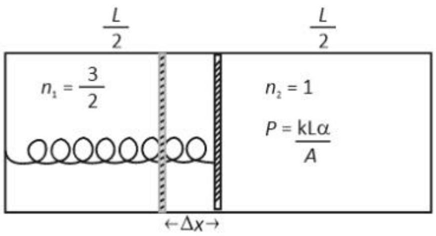
= 2.33

13. The left and right compartments of a thermally isolated container of length L are separated by a thermally conducting, movable piston of area A. The left and right compartments are filled with $\frac{3}{2}$ and 1 moles of an ideal gas, respectively. In the left compartment the piston is attached by a spring with spring constant k and natural length $\frac{2L}{5}$ In thermodynamic equilibrium, the piston is at a distance $\frac{L}{2}$ from the left and right edges of the container as shown in the figure. Under the above conditions, if the pressure in the right compartment is $P=\frac{kL}{A}\alpha$, then the value of α is _____



Answer (00.20)

Sol.

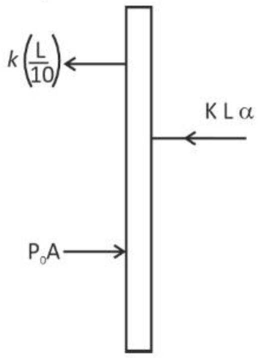


$\Delta x=\frac{L}{2}-\frac{2L}{5}=\frac{L}{10}$,

$pv=nRT$

$P_0=\frac{3}{2}\frac{kl\alpha}{A}$

On piston



$$P_0A = \frac{3}{2}KL\alpha$$

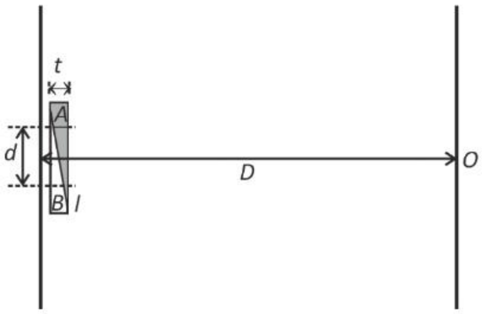
At equilibrium

$$\frac{KL}{10} - \frac{3}{2}KL\alpha = -KL\alpha$$

$$\alpha = \frac{2}{10} = 0.2$$

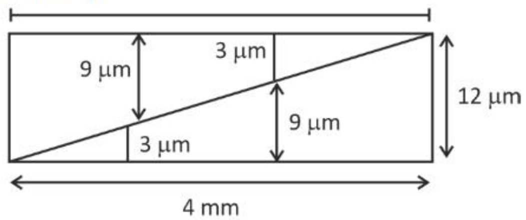
$\Rightarrow$ 00.20

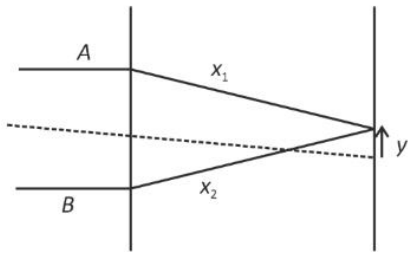
14. In a Young's double slit experiment, a combination of two glass wedges A and B, having refractive indices 1.7 and 1.5, respectively, are placed in front of the slits, as shown in the figure. The separation between the slits is d = 2 mm and the shortest distance between the slits and the screen is D = 2 m. Thickness of the combination of the wedges is t = 12 μm. The value of l as shown in the figure is 1 mm. Neglect any refraction effect at the slanted interface of the wedges. Due to the combination of the wedges, the central maximum shifts (in mm) with respect to O by ____



Answer (01.20)

Sol.





$(x_1)_{extra} = \left[(1.5-1)^3 + (1.7-1)9\right] \times 10^{-6}$

$= (1.5 + 6.3) \times 10^{-6}$

$= 7.8 \times 10^{-6}$

$(x_2)_{extra} = [(1.5 - 1)\,9 + (1.7 - 1)3] \times 10^{-6}$

$= (4.5 + 2.1) \times 10^{-6}$

$= 6.6 \times 10^{-6}$

$\Delta x = 1.2 \times 10^{-6}$

$$\frac{d}{D} y = 1.2 \times 10^{-6}$$

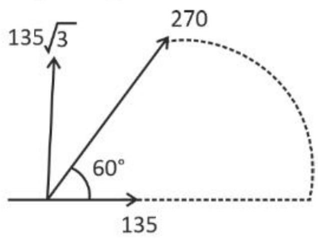
$$y = \frac{1.2 \times 10^{-6} \times 2}{2 \times 10^{-3}}$$

$= 1.2$ mm

15. A projectile of mass 200 g is launched in a viscous medium at an angle 60° with the horizontal, with an initial velocity of 270 m/s. It experiences a viscous drag force $\vec{F} = -c\vec{v}$ where the drag coefficient $c = 0.1$ kg/s and $\vec{v}$ is the instantaneous velocity of the projectile. The projectile hits a vertical wall after 2 s. Taking $e = 2.7$, the horizontal distance of the wall from the point of projection (in m) is

Answer (170.00)

Sol.



$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{F} = 0.1\,\text{kg}(v_x\hat{i} + v_y\hat{j}) - 10m\hat{j}$$

$$\vec{a} = -\left(\frac{v_x\hat{i} + v_y\hat{j}}{2}\right) - 10\hat{j}$$

$$\vec{a} = -\frac{v_x}{2}\hat{i} - \left(\frac{v_y + 20}{2}\right)\hat{j}$$

$$\frac{dv_x}{dt} = \frac{-v_x}{2}$$

$$\int_{135}^{v} \frac{dv_x}{v_x} = -\int_{0}^{t} \frac{dt}{2}$$

$$\ln\left(\frac{v}{135}\right) = -\frac{t}{2}$$

$$v = 135e^{\frac{-t}{2}}$$

$$\frac{dx}{dt} = 135e^{\frac{-t}{2}}$$

$$\int dx = 135\int_{0}^{2} e^{-t/2}$$

$$= 135(-2)\,[-1 + e^{-1}]$$

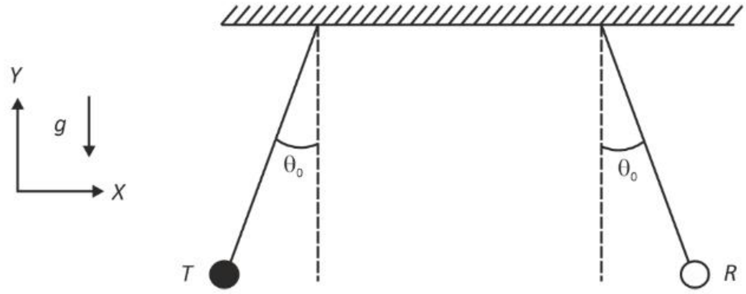
$$= 270\left(1 - \frac{1}{e}\right)$$

$$= 270\left(1 - \frac{1}{2.7}\right)$$

$$= 270 \times \frac{1.7}{2.7}$$

$$= 170$$

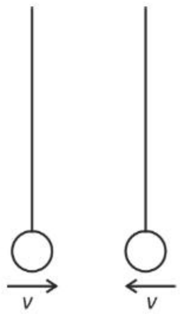
16. An audio transmitter (T) and a receiver (R) are hung vertically from two identical massless strings of length 8 m with their pivots well separated along the X axis. They are pulled from the equilibrium position in opposite directions along the X axis by a small angular amplitude $\theta_0 = \cos^{-1}(0.9)$ and released simultaneously. If the natural frequency of the transmitter is 660 Hz and the speed of sound in air is 330 m/s, the maximum variation in the frequency (in Hz) as measured by the receiver (Take the acceleration due to gravity g = 10 m/s^2) is ____



Answer (32.00)

Sol. $L = 8$ m $\theta_0 = \cos^{-1}(0.9)$ $f_0 = 660$ Hz

$V_s = 330$ g = 10 m/s^2



$$v = \sqrt{2gl(1-\cos\theta)}$$

$$= \sqrt{2\times10\times8\times0.1} = 4 \text{ m/s}$$

$$f_{max} = f_0\left(\frac{v_s+v}{v_s-v}\right)$$

$$f_{min} = f_0\left(\frac{v_s-v}{v_s+v}\right)$$

$$f_{max} - f_{min} = f_0\left(\frac{v_s+v}{v_s-v} - \frac{v_s-v}{v_s+v}\right)$$

$$= f_0\left(\frac{4\times v_s\times v}{(v_s^2-v^2)}\right) = \frac{660\times4\times330\times4}{(330^2-4^2)} \approx 32\text{Hz}$$

$$\boxed{\Delta f = 32 \text{ Hz}}$$

PART-III : CHEMISTRY

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	:	+3	If **ONLY** the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

1. During sodium nitroprusside test of sulphide ion in an aqueous solution, one of the ligands coordinated to the metal ion is converted to

(A) NOS^-

(B) SCN^-

(C) SNO^-

(D) NCS^-

Answer (A)

Sol. Sodium nitroprusside test is used for detection of sulphur in organic compound.

$$\underset{\text{(Sodium extract)}}{Na_2S} + \underset{\text{Sodium nitroprusside}}{Na_2[Fe(CN)_5NO]} \longrightarrow \underset{\text{Violet colour}}{Na_4[Fe(CN)_5NOS]}$$

$\overset{+}{NO}$ ligand is converted into NOS^- ligand.

2. The complete hydrolysis of ICl, ClF_3 and BrF_5, respectively, gives

(A) IO^-, ClO_2^- and BrO_3^-

(B) IO_3^-, ClO_2^- and BrO_3^-

(C) IO^-, ClO^- and BrO_2^-

(D) IO_3^-, ClO_4^- and BrO_2^-

Answer (A)

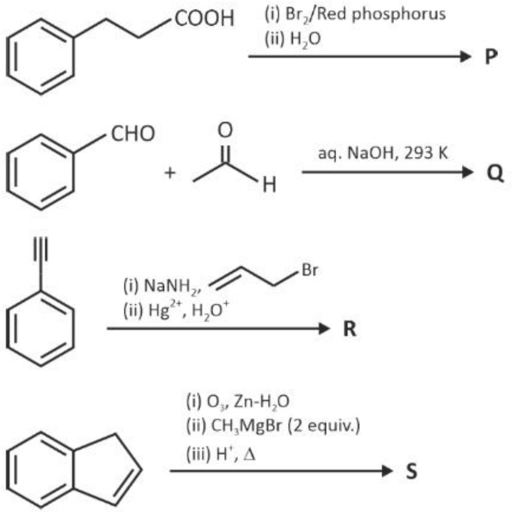
Sol. $ICl + H_2O \rightarrow HCl + HIO\ (IO^-)$

$BrF_5 + 3H_2O \rightarrow 5HF + HBrO_3\ (BrO_3^-)$

$ClF_3 + 2H_2O \rightarrow HClO_2 + 3HF\ (ClO_2^-)$

$\therefore\ IO^-, ClO_2^-, BrO_3^-$

3. Monocyclic compounds **P**, **Q**, **R** and **S** are the major products formed in the reaction sequences given below.



The product having the highest number of unsaturated carbon atom(s) is

(A) P

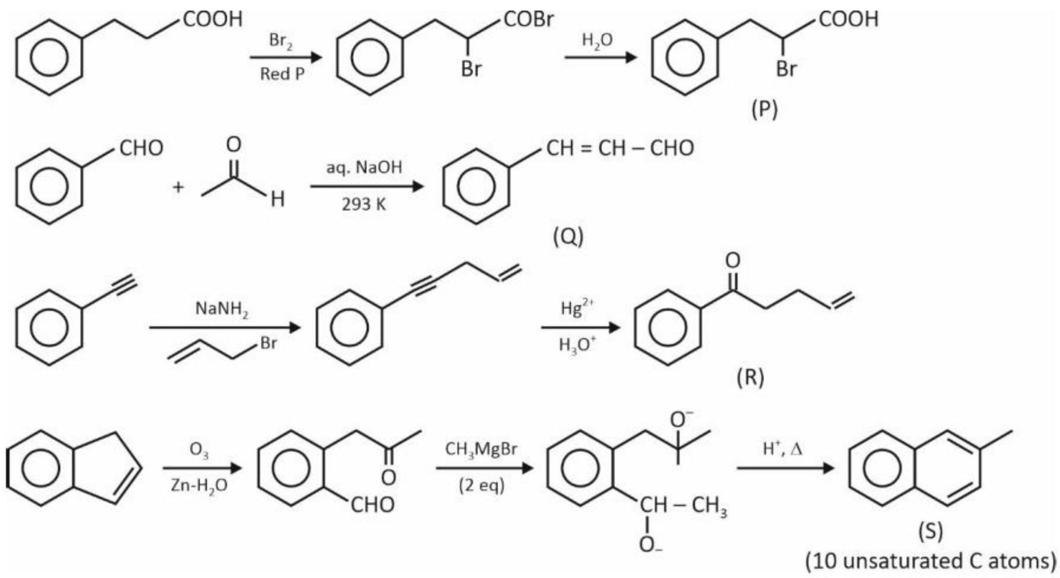
(B) Q

(C) R

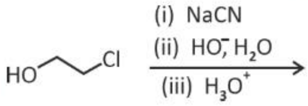
(D) S

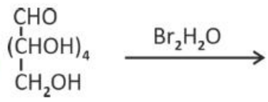
Answer (D)

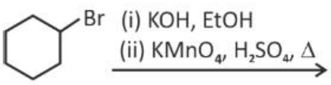
Sol.

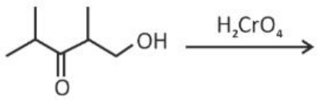


4. The correct reaction/reaction sequence that would produce a dicarboxylic acid as the major product is

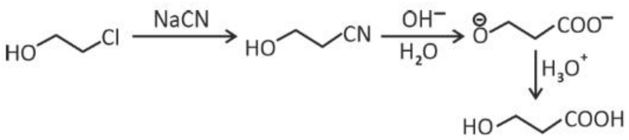
(A) 

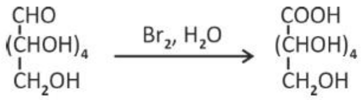
(B) 

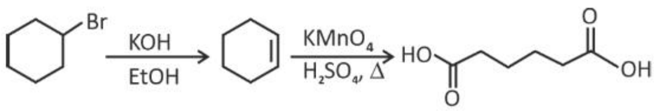
(C) 

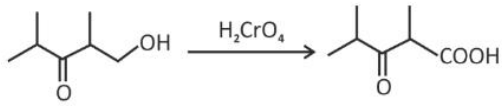
(D) 

Answer (C)

Sol. 







SECTION 2 (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks	:	+4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	:	+2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

5. The correct statement(s) about intermolecular forces is(are)

(A) The potential energy between two point charges approaches zero more rapidly than the potential energy between a point dipole and a point charge as the distance between them approaches infinity.

(B) The average potential energy of two rotating polar molecules that are separated by a distance r has $1/r^3$ dependence.

(C) The dipole-induced dipole average interaction energy is independent of temperature.

(D) Nonpolar molecules attract one another even though neither has a permanent dipole moment.

Answer (D)

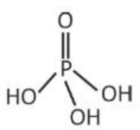
Sol.
- Potential energy between two point charges varies as $\frac{1}{r}$ and between point charge and dipole varies as $\frac{1}{r^2}$, So, $\frac{1}{r} \to 0$ more slowly than $\frac{1}{r^2} \to 0$. [(A) $\to$ incorrect]
- The instantaneous interaction between molecules (rotating polar molecules) depends on orientation, the dipole-dipole interaction energy has $\frac{1}{r^6}$ dependence. [(B) $\to$ incorrect]
- Dipole induced dipole involves a permanent dipole inducing a dipole in a non-polar molecule. This interaction is a typed of Vander Waals force involving non polar molecule and hence depends on temperature. [(C) $\to$ incorrect]
- Due to London dispersion forces, temporary induced dipole moments occur and hence non polar molecules can attract each other. [(D) $\to$ correct]

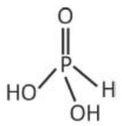
6. The compound(s) with P-H bond(s) is(are)

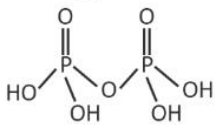
(A) H_3PO_4 (B) H_3PO_3

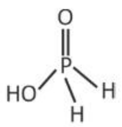
(C) $H_4P_2O_7$ (D) H_3PO_2

Answer (B, D)

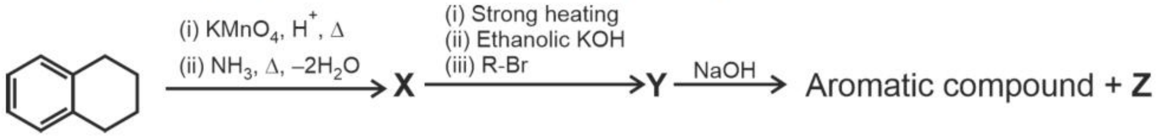
Sol. H_3PO_4 :


H_3PO_3 :


$H_4P_2O_7$:


H_3PO_2 :


7. For the reaction sequence given below, the correct statement(s) is(are)



(A) Both **X** and **Y** are oxygen containing compounds

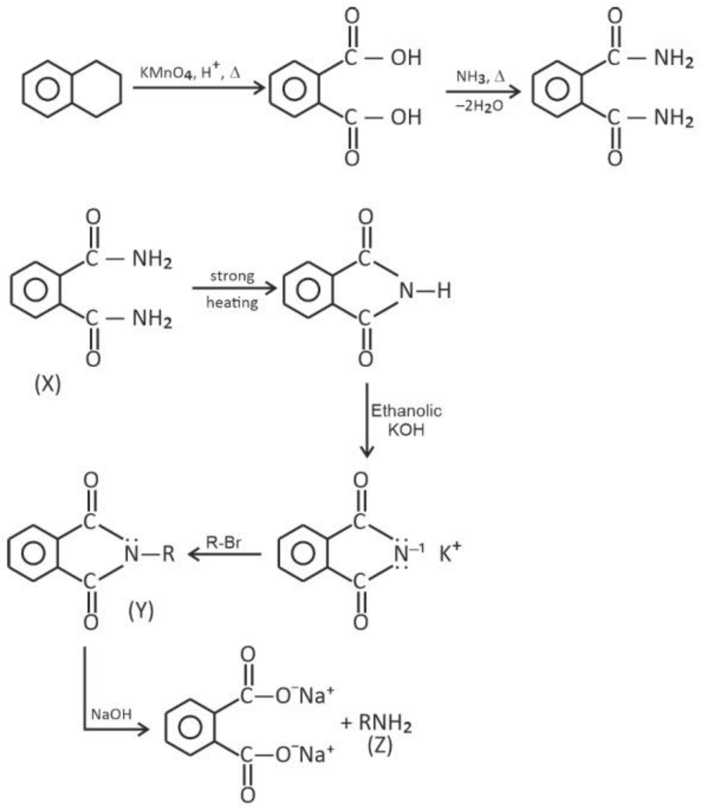
(B) **Y** on heating with $CHCl_3$/KOH forms isocyanide

(C) **Z** reacts with Hinsberg's reagent

(D) **Z** is an aromatic primary amine

Answer (A, B, C)

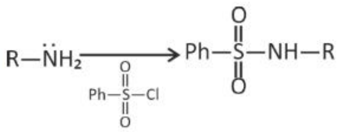
Sol.



(A) Option (A) is correct.

(B) $RNH_2 \xrightarrow{CHCl_3/KOH} RNC$

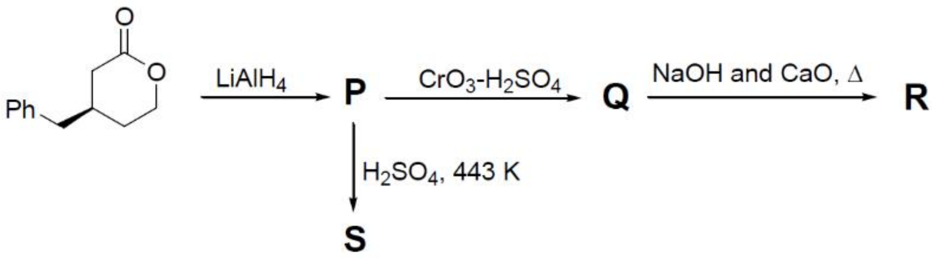
Option (B) is correct.

(C) 

Option (C) is correct.

(D) Option (D) is incorrect.

8. For the reaction sequence given below, the correct statement(s) is(are)



(A) **P** is optically active

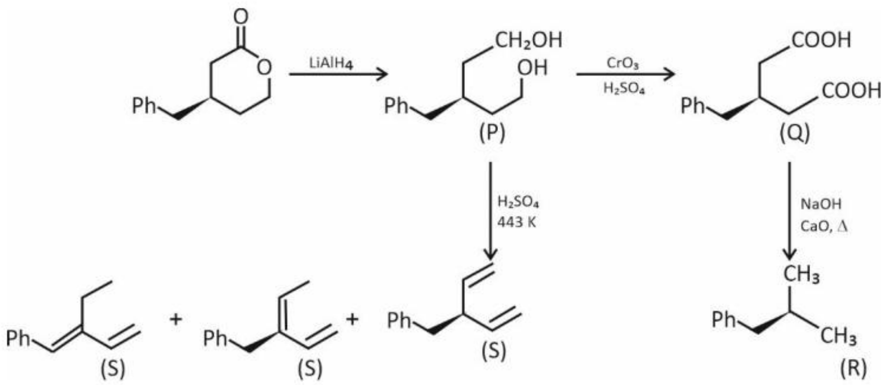
(B) **S** gives Bayer's test

(C) **Q** gives effervescence with aq. $NaHCO_3$

(D) **R** is an alkyne

Answer (B, C)

Sol.



(A) P is optically inactive

(B) S is Alkene, and Alkene gives Bayer's test

(C) Q has carboxylic acid and it gives effervescence with $NaHCO_3$

(D) R is Not Alkyne

SECTION 3 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

9. The density (in g cm^{-3}) of the metal which forms a cubic close packed (ccp) lattice with an axial distance (edge length) equal to 400 pm is ________.

Use: Atomic mass of metal = 105.6 amu and Avogadro's constant = 6×10^{23} mol^{-1}

Answer (11.00)

Sol. Density = $\dfrac{ZM}{N_A(a)^3}$

$$= \frac{4 \times 105.6}{6 \times 10^{23}\left(4 \times 10^{-8}\right)^3} \text{ g cm}^{-3}$$

$= 11.00$ g cm^{-3}

10. The solubility of barium iodate in an aqueous solution prepared by mixing 200 mL of 0.010 M barium nitrate with 100 mL of 0.10 M sodium iodate is $X \times 10^{-6}$ mol. dm^{-3}. The value of X is ________.

Use : Solubility product constant (K_{sp}) of barium iodate = 1.58×10^{-9}

Answer (3.95)

Sol. $Ba(NO_3)_2(aq) + 2NaIO_3(aq) \rightleftharpoons Ba(IO_3)_2(s) + 2NaNO_3(aq)$

2 m mol	10 m mol	—	—
—	6 m mol	2 m mol	4 m mol

Left $[IO_3^-]_{eqm} = \left(\dfrac{6}{300}\right) M = 0.02M$

$K_{sp} = [Ba^{2+}][IO_3^-]^2$

$[Ba^{2+}] = \dfrac{1.58 \times 10^{-9}}{(0.02)^2} = 3.95 \times 10^{-6}$ M

Solubility of $Ba(IO_3)_2$ = X = 3.95×10^{-6} M

$\therefore$ X = 3.95

11. Adsorption of phenol from its aqueous solution on to fly ash obeys Freundlich isotherm. At a given temperature, from 10 mg g^{-1} and 16 mg g^{-1} aqueous phenol solutions, the concentrations of adsorbed phenol are measured to be 4 mg g^{-1} and 10 mg g^{-1}, respectively. At this temperature, the concentration (in mg g^{-1}) of adsorbed phenol from 20 mg g^{-1} aqueous solution of phenol will be ______.

Use : $\log_{10} 2 = 0.3$

Answer (15.62 − 16.00)

Sol. $\frac{x}{m} = K(C)^{\frac{1}{n}}$

$4 = K\ (10)^{\frac{1}{n}}$ ... (i)

$10 = K\ (16)^{\frac{1}{n}}$ ...(ii)

From (ii) and (i)

$\log 4 - \log 10 = \frac{1}{n}\ (\log 10 - \log 16)$

$2\log 2 - 1 = \frac{1}{n}(1 - 4\log 2)$

$0.6 - 1 = \frac{1}{x}(1 - 1.2)$

$-0.4 = \frac{-0.2}{n}$

$n = \frac{2}{4} = \left(\frac{1}{2}\right)$

Now $10 = K\ (16)^2$ or $4 = K\ (10)^2$

$K = \frac{10}{256}$ $K = \frac{4}{100}$

So, $\frac{x}{m} = K(20)^2$ $\frac{x}{m} = K(20)^2$

$\frac{x}{m} = \frac{10}{256} \times 400$ $\frac{x}{m} = \frac{4}{100} \times 400$

$\frac{x}{m} = 15.625$ $\frac{x}{m} = 16$

12. Consider a reaction A + R $\rightarrow$ *Product*. The rate of this reaction is measured to be k[A][R]. At the start of the reaction, the concentration of R, $[R]_0$, is 10-times the concentration of *A*, $[A]_0$. The reaction can be considered to be a pseudo first order reaction with assumption that k[R] = k′ is constant. Due to this assumption, the relative error (in %) in the rate when this reaction is 40% complete is ______.

[k and k′ represent corresponding rate constants]

Answer (04.17)

Sol. A + R $\rightarrow$ Product

r = k[A] [R]

Now, A + R $\rightarrow$ Product

A_0 $10A_0$

$(1 - 0.4)\ A_0$ $(10 - 0.4)A_0$

$(0.6\ A_0)\ (9.6\ A_0)$

$r = k[0.6\ A_0]\ [9.6\ A_0]$ ...(1)

Again, A + R $\rightarrow$ Product

$r' = k'[A]$ $k' = (10A_0)k$

Now, A + R $\rightarrow$ Product

A_0

$(1 - 0.4)A_0$

$0.6\ A_0$

$r' = k(10\ A_0)\ (0.6\ A_0)$

$$\text{Relative error} = \frac{6k(A_0)^2 - k(0.6\times 9.6)(A_0)^2}{k(0.6\times 9.6)A_0^2}\times 100 = 4.17\%$$

13. At 300 K, an ideal dilute solution of a macromolecule exerts osmotic pressure that is expressed in terms of the height (h) of the solution (density = 1.00 g cm^{-3}) where h is equal to 2.00 cm. If the concentration of the dilute solution of the macromolecule is 2.00 g dm^{-3}, the molar mass of the macromolecule is calculated to be $\boldsymbol{X} \times 10^4$ g mol^{-1}. The value of $\boldsymbol{X}$ is ________.

Use: Universal gas constant (R) = 8.3 JK^{-1} mol^{-1} and acceleration due to gravity (g) = 10 ms^{-2}

Answer (02.49)

Sol. Density of solution = 1.00 g cm^{-3} = 1000 kg m^{-3}

Height h = 2 cm = 2×10^{-2} m

Acceleration due to gravity g = 10 ms^{-2}

Osmotic pressure $\pi = h\rho g$

$= 2 \times 10^{-2} \times 1000 \times 10$

$= 200$ N m^{-2}

Also, $\pi = cRT$

$$200 = \left(\frac{2000}{M}\right) 8.3 \times 300$$

M = 24900 g mol^{-1}

= 2.49×10^4 g mol^{-1}

$\therefore$ X = 2.49

14. An electrochemical cell is fueled by the combustion of butane at 1 bar and 298 K. Its cell potential is $\frac{\boldsymbol{X}}{F} \times 10^3$ volts, where F is the Faraday constant. The value of $\boldsymbol{X}$ is _______.

Use: Standard Gibbs energies of formation at 298 K are: $\Delta_f G^o_{CO_2} = -394\ kJ\ mol^{-1}$; $\Delta_f G^o_{water} = -237\ kJ\ mol^{-1}$; $\Delta_f G^o_{butane} = -18\ kJ\ mol^{-1}$

Answer (105.50)

Sol. $C_4H_{10} + 6.5O_2 \rightarrow 4CO_2 + 5H_2O$

$\Delta G^o_{reaction} = (-4 \times 394) + (-5 \times 237) - (-18) = -2743\ kJ$

Now, $E^o = \frac{-1}{26\ F} \times \Delta G^o$

$\frac{X}{F} \times 10^3 = -\frac{1}{26\ F} \times [-2743 \times 10^3]$

$X = \frac{2743}{26} = 105.50$ volts

15. The sum of the spin only magnetic moment values (in B.M.) of $[Mn(Br)_6]^{3-}$ and $[Mn(CN)_6]^{3-}$ is _______.

Answer (07.73)

Sol. $\left[Mn(Br_6)\right]^{3-} \Rightarrow Br^- \Rightarrow$ WFL

$P > \Delta_o$

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad \underline{1}\ \ \underline{\ \ }\ \ e_g \\ Mn(+3) \Rightarrow 3d^4 \rightarrow \underline{1}\ \underline{1}\ \underline{1}\ \ t_{2g} \end{array}$$

$\left[Mn(Br)_6\right]^{3-} \Rightarrow sp^3d^2$

$\therefore\ n = 4$

$\mu = \sqrt{4(4+2)} = \sqrt{24}$ BM

$= 4.90$ BM

$\left[Mn(CN)_6\right]^{3-}$

$CN^- \Rightarrow$ SFL

$P < \Delta_o$

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad \underline{\ \ }\ \ \underline{\ \ } \\ Mn(+3) \Rightarrow 3d^4 \rightarrow \underline{1\!\!\downarrow}\ \underline{1}\ \underline{1} \end{array}$$

$\left[Mn(CN)_6\right]^{3-} \Rightarrow d^2sp^3$

n = 2

$\mu = \sqrt{2(2+2)}$ BM

$= \sqrt{8}$ BM

= 2.83 BM

$\therefore$ Sum = 4.90 + 2.83 = 7.73 BM

16. A linear octasaccharide (molar mass = 1024 g mol^{-1}) on complete hydrolysis produces three monosaccharides: ribose, 2-deoxyribose and glucose. The amount of 2-deoxyribose formed is 58.26% (w/w) of the total amount of the monosaccharides produced in the hydrolyzed products. The number of ribose unit(s) present in one molecule of octasaccharide is _______.

Use: Molar mass (in g mol^{-1}): ribose = 150, 2-deoxyribose = 134, glucose = 180; Atomic mass (in amu): H = 1, O = 16

Answer (02.00)

Sol. $\text{Octasaccharide} + 7H_2O \longrightarrow \text{Ribose} + \underset{(58.26\%)}{\text{2-deoxyribose}} + \text{Glucose}$

Total mass at reactant side = 1024 + (7 × 18) = 1024 + 126 = 1150 g

Mass of 2-deoxyribose formed = $1150 \times \frac{58.26}{100} = 669.99 \approx 670$ g

Total units of 2-deoxyribose = $\frac{670}{134}$ = 5 units

Possible unit of glucose produced = 1

Possible unit of ribose = 2

So that total mass of product = 670 + 180 + (2 × 150) = 1150 g

So the answer is 02.00